Multivariable Calculus

Quiz 11 **SOLUTIONS**

1) Compute the line integral

$$\int_{\mathcal{C}} \left(x^2 + z\right)\ ds$$

where $\mathcal{C}$ is the helix $\vec{r}(t) = \langle \cos(t), \sin(t), t\rangle$ on $0 \leq t \leq \pi$.

Solution:

$$\begin{aligned}
\vec{r}\,'(t) &= \langle -\sin(t), \cos(t), 1\rangle \\
\|\vec{r}\,'(t)\| &= \sqrt{\sin^2(t) + \cos^2(t) + 1} = \sqrt{2} \\
\int_{\mathcal{C}} \left(x^2 + z\right)\ ds &= \int_0^{\pi} \left(x^2(t) + z(t)\right) \|\vec{r}\,'(t)\|\ dt \\
&= \int_0^{\pi} \left(\cos^2(t) + t\right) \sqrt{2}\ dt \\
&= \sqrt{2}\int_0^{\pi} \cos^2(t)\ dt + \sqrt{2}\int_0^{\pi} t\ dt \\
&= \frac{\sqrt{2}}{2}\int_0^{\pi} \left(1 + \cos(2t)\right)\ dt + \frac{\sqrt{2}}{2}\left(t^2\Big|_0^{\pi}\right) \\
&= \frac{\sqrt{2}}{2}\left(\pi^2 + \pi\right)
\end{aligned}$$

TURN OVER

2) Compute the line integral $\int_{\mathcal{C}} \vec{F} \cdot d\vec{r}$ for the vector field

$$\vec{F} = \langle x^2 y^3,\ -y\sqrt{x} \rangle$$

over the curve $\mathcal{C}$ given by $\vec{r}(t) = \langle t^2, -t^3 \rangle$ on $0 \le t \le 1$.

Solution:

$$\begin{aligned}
\vec{r}\,'(t) &= \left\langle 2t,\ -3t^2 \right\rangle \\
\int_{\mathcal{C}} \vec{F} \cdot d\vec{r} &= \int_{\mathcal{C}} \vec{F}(\vec{r}(t)) \cdot \vec{r}\,'(t)\ dt \\
&= \int_0^1 \left\langle \left(t^2\right)^2 \left(-t^3\right)^3,\ -(-t^3)\sqrt{t^2} \right\rangle \cdot \left\langle 2t,\ -3t^2 \right\rangle\ dt \\
&= \int_0^1 \left(-2t^{14} - 3t^6\right)\ dt \\
&= -\frac{2t^{15}}{15} - \frac{3t^7}{7}\Bigg|_0^1 \\
&= -\frac{59}{105}
\end{aligned}$$